Guten Morgen gA (i)

20.3.18

2014

A1 a) Ansatz $f(x) = 0$

$f(x) = (1+x)e^x$

$0 = (1+x) \cdot \underbrace{e^x}_{\neq 0}$

$0 = 1+x \mid -1$

$-1 = x$

b) Ansatz $F'(x) = f(x)$

$F(x) = x \cdot e^x$

$F'(x) = 1 \cdot e^x + x \cdot e^x$

$= e^x(1+x)$

$= f(x)$ ✓

Nullproduktsatz

b) 2. Teil:

$g(x) = x e^x + C$

$g(0) = e$

$e = \underbrace{0 \cdot e^0}_{0} + C$

$e = C$

$g(x) = x e^x + e$

Produktregel $f = u \cdot v$

$f' = u' \cdot v + u v'$

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2014 P2

a) $f(x) = -x^2 + 1$

orientierte Flächeninhalt

b) $\int_{-1}^{1} (-x^2 + a)\,dx = 0$

$\int_{-1}^{1} (-x^2 + a)\,dx = \left[-\frac{1}{3}x^3 + ax\right]_{-1}^{1}$

$= -\frac{1}{3} + a - \left(-\frac{1}{3}\cdot(-1)^3 + a\cdot(-1)\right)$

$= -\frac{1}{3} + a \quad -\frac{1}{3} + a = -\frac{2}{3} + 2a$

$-\frac{2}{3} + 2a = 0 \quad /+\frac{2}{3}$

$2a = \frac{2}{3} \quad /:2$

$a = \frac{1}{3}$

c) 6 Infusion:

$\lim_{t\to\infty} \left(50\cdot(1 - e^{-0{,}02t})\right) = 50$

$e^{-\infty} = 0$

Insekt: $m(t) = 45$ $99\cdot 50 = 45$ insect

$x = 115{,}13$ min

Insekt $m'(t) = 95$ u Derive insect

$t = 34{,}66 \approx 35$ min

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6 Infusion

e) $\int_0^{10} \underbrace{e^{-0,02t}}_{u'(t)} dt = \left[\cancel{-\frac{1}{0,02}} \cancel{e^{-0,02t}} \right]_0^{10}$

$f(x) = a \cdot e^{v}$

$F(x) = \frac{a}{v'} \cdot e^{v} \quad (+c)$

$= \left[\underbrace{50 \left(1 - e^{-0,02t}\right)}_{u(t)} \right]_0^{10}$

$\approx 9,06 \text{ mg}$

$\left[F(x)\right]_a^b = F(b) - F(a)$

f) $m(300) = 49,88 \text{ mg}$

$5 \cdot 60$

$a = 49,88$

$t_H = 360 \text{ min}$

$f(t) = a \cdot e^{kt}$

Halbwertszeit $e^{kt} = 0,5$ oder 2

Halb

Verdopplungszeit

$0,5 = e^{k \cdot 360} \quad | \ln$

$\ln 0,5 = k \cdot 360 \quad | :360$

$k \approx -0,1155$

$f(t) = 49,88 \, e^{-0,1155t}$

c) $f(x) = 10^{-3}$ intersect

$t \approx 93{,}66$ min

≈ 94 min

4. Bd.:

$V = Q \cdot 100$

$Q = \int_{-25}^{25} f(x)\,dx \approx 327{,}16$

$V \approx 32716\ m^3 > 30000\ m^3$ ✓

2010 1 t

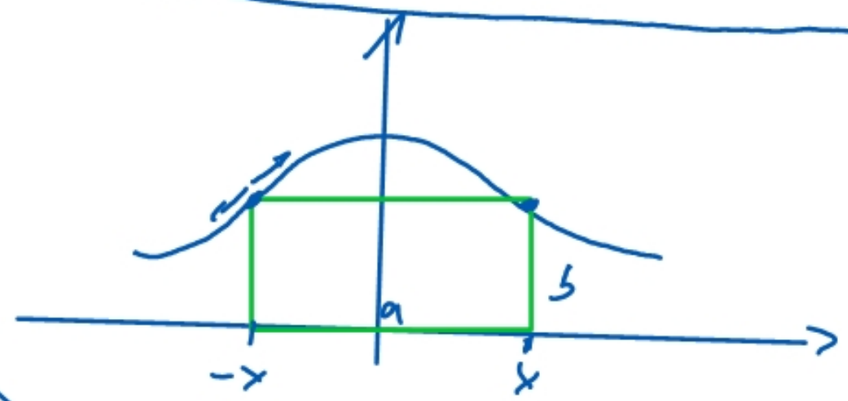
a) 1. Bed $f(0) = 10$ ✓

2. Bd $f(\pm 25) > 2$ ✓

3 Bed b in Höhe 5 m bestimmen

$f(x) = 5$ $x_1 = -16{,}65$, $x_2 = 16{,}65$ } addieren >30 ✓

b)



$a = 2x$

$b = f(x)$ $A = a \cdot b$

$A(x) = 2x \cdot f(x)$

↓

Max (14,14 | 171,55)

2010 1*

c)

F	f	N	E	W	
			N	E	W
f	f'				
			N	E	W
f'	f''				

f' = Steigung

x-Achse Asymptote → f' → 0

f = HP → f': —

d) $f_a(x) = 10 e^{-ax^2}$

$f'_a(x) = -20ax \, e^{-ax^2}$

Ansatz $f''_a(x) = 0$

$0 = (40a^2x^2 - 20a) \underbrace{e^{-ax^2}}_{\neq 0}$

Achse ≙ y-Koordinate

$0 = 40a^2x^2 - 20a \quad | +20a$

$20a = 40a^2x^2 \quad | : (40a^2)$

$\frac{0{,}5}{a} = x^2 \quad \pm\sqrt{}$

$\sqrt{\frac{0{,}5}{a}} = x \approx 1{,}2$

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$$\pm\sqrt{\frac{0{,}5}{\alpha}} = \pm\sqrt{\frac{1}{2\alpha}} = x_{1,2}$$

$$f_\mu\left(\pm\sqrt{\frac{1}{2\alpha}}\right) = 10 \cdot e^{-\alpha\left(\pm\sqrt{\frac{1}{2\alpha}}\right)^2}$$

$$= 10\, e^{-\alpha \cdot \frac{1}{2\alpha}}$$

$$= 10\, e^{-\frac{1}{2}}$$ unabhängig von α $\rightarrow$ alle auf derselben Höhe

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2010 1B

~~2010~~ 1B

a) Ansatz:

$f''(2) = 0$ ✓

$f'''(2) \neq 0$ ✓

$f(2) = 2$ ✓

$f'(-2) = -4{,}5$

$f'(6) = -4{,}5$

$f'(+2) = 1{,}5$

graphisch oder tabellarische Steigung

betragsmäßig graphische Steigung

b) $f(x) = \frac{1}{8}x^3 + \frac{3}{4}x^2$

Ansatz $f(x) = g(x)$

$x_1 = -2$

$x_2 = 2$

$x_3 = 6$

$F(x) = -\frac{1}{32}x^4 + \frac{1}{4}x^3$

$$A = \int_{-2}^{6} |f(x) - g(x)| \, dx$$

$= 16$ FE

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2018 1B

c) $f_k(x) = -\frac{1}{8}x^3 + \frac{3k}{8}x^2$

$f_k'(x) = -\frac{3}{8}x^2 + \frac{3k}{4}x$

$f_k''(x) = -\frac{3}{4}x + \frac{3k}{4}$

$f_k'''(x) = -\frac{3}{4} = 0$

$W(k \mid \frac{1}{4}k^3)$

1. $k = x$
2. $y = \frac{1}{4}k^3$

einsetzen $\boxed{y = \frac{1}{4}x^3} = h(x)$

Ansatz:

$h'(k) = 2 \cdot f_k'(k)$

Ansatz $f_k''(x) = 0$

$0 = -\frac{3}{4}x + \frac{3k}{4} \quad | -\frac{3k}{4}$

$-\frac{3k}{4} = -\frac{3}{4}x \quad | \cdot (-\frac{4}{3})$

$\underline{\underline{k = x}}$

$f_k(k) = -\frac{1}{8}k^3 + \frac{3k}{8} \cdot k^2 = -\frac{1}{8}k^3 + \frac{3}{8}k^3$

$= \frac{1}{4}k^3$

$h'(x) = \frac{3}{4}x^2 \qquad h'(k) = \frac{3}{4}k^2 = 2 \cdot \frac{3}{8}k^2$

$f_k'(x) = -\frac{3}{8}x^2 + \frac{3k}{4} \cdot x = -\frac{3}{8}x^2 + \frac{3}{4}k^2$

$= \frac{3}{8}k^2$

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d) Ansatz: $f'_{2t}(x) = 0$

$$0 = -\frac{3}{8}x^2 + \frac{3t}{4}x$$

$$0 = x\left(-\frac{3}{8}x + \frac{3t}{4}\right)$$

$x_1 = 0$

$$0 = -\frac{3}{8}x + \frac{3t}{4} \quad | -\frac{3t}{4}$$

$$-\frac{3t}{4} = -\frac{3}{8}x \quad | \cdot\left(-\frac{8}{3}\right)$$

$2t \Rightarrow$

$$f_t(2t) = -\frac{1}{8}(2t)^3 + \frac{3t}{8}\cdot(2t)^2$$

$$= -t^3 + \frac{3}{2}t^3 = 0{,}5\,t^3$$

$H(2t \mid 0{,}5\,t^3)$

$O(0 \mid 0)$

$$m = \frac{\frac{1}{4}t^3 - 0}{t - 0} = \frac{1}{4}t^2$$

Ursprungsgerade

$g(x) = m\,x$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$g(x) = \frac{1}{4}t^2 x$

$$g(2t) = \frac{1}{4}t^2 \cdot 2t = \frac{1}{2}t^3$$

$$f''(0) = \frac{3t}{4} > 0 \Rightarrow TP$$

$$f''_t(2t) = -\frac{3}{4}\cdot 2t + \frac{3t}{4} = -\frac{3}{2}t + \frac{3}{4}t = -\frac{3}{4}t < 0 \Rightarrow HP$$

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2017 1A

a)

$N(t) = 1000000\, e^{-0{,}25\,t}$

Ansatz $N(30) \approx 553$ Erreger

Ansatz $N(t) = 100 \rightarrow t \approx 36{,}8 \approx 37$ min

$N(t) > 0 \rightarrow$ niemals stirbt?

Ansatz: $250000 = 1000000\, e^{-kt} \quad | : 1000000$

$0{,}25 = e^{-kt}$ GTR intersect

$k \approx 0{,}92$

b)

$N'(t) = N_0 \cdot (-k)\, e^{-kt}$

$= -k \cdot N_0 \cdot e^{-kt}$

$N'(t) = -k \cdot 2 \cdot N_0\, e^{-kt}$

$= 2 \cdot (-k) \cdot N_0 \cdot e^{-kt} = 2 \cdot N'(t)$

Ansatz $N(D) = \frac{1}{10} N_0$

$\frac{1}{10} N_0 = N_0\, e^{-k \cdot D} \quad | : N_0$

$\frac{1}{10} = e^{-k \cdot D} \quad | \ln(\,)$

$\ln\left(\frac{1}{10}\right) = -k \cdot D \quad | \cdot \left(-\frac{1}{k}\right)$

$-\frac{1}{k} \cdot \ln\left(\frac{1}{10}\right) = D$

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$N(30)$

$$\int_0^{30} N'(t)\,dt = \left[N(t)\right]_0^{30}$$

$$= N(30) - N(0)$$

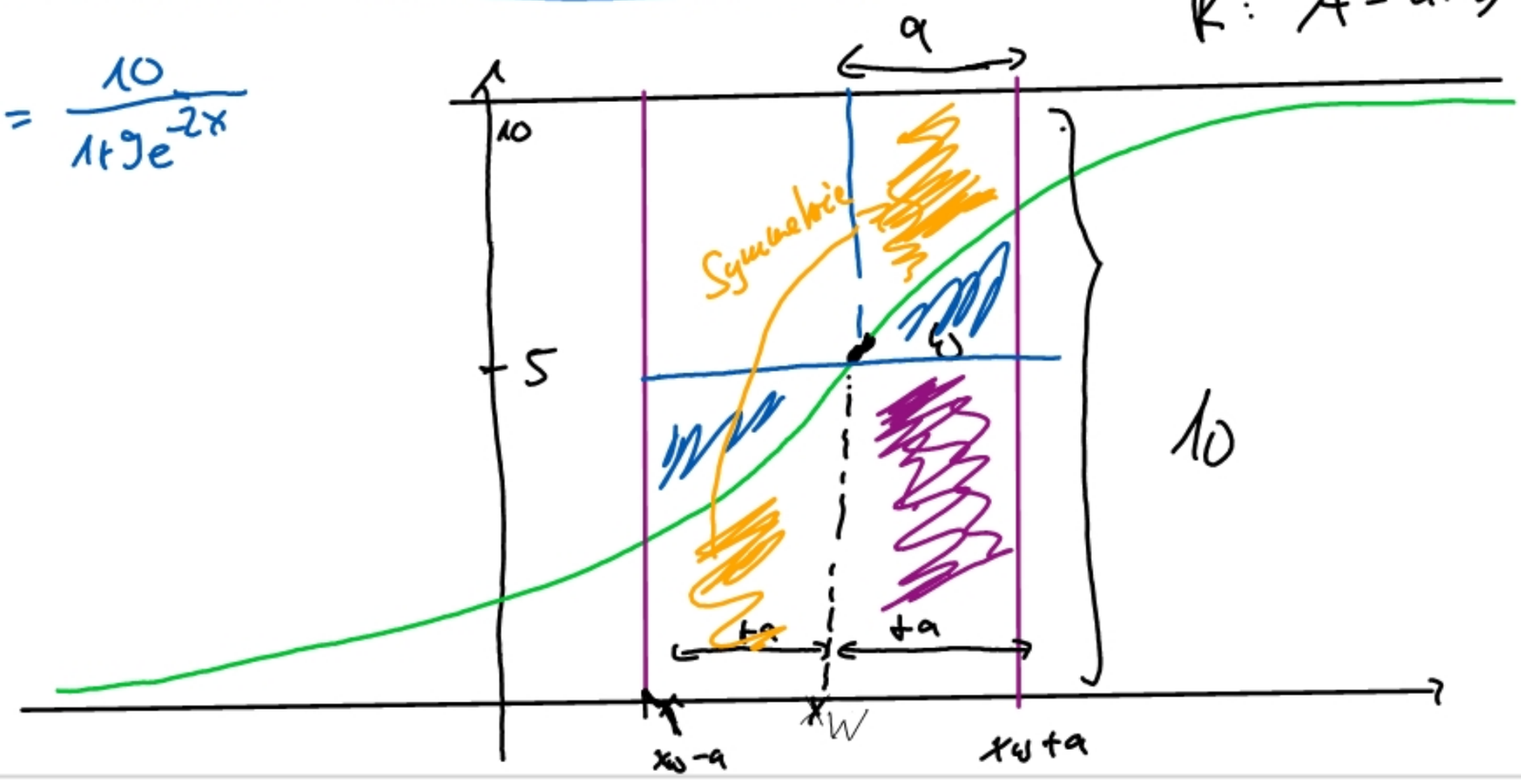
Unterschied sich um $-N(0)$

Anfangswert / Bestand

$$A = a \cdot 10$$

R: $A = a \cdot 5$

$$f(x) = \frac{10}{1 + 9e^{-2x}}$$



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2017 A1B

a)

$\frac{20m}{80m} = \frac{1}{4} < \frac{1}{3}$ ✓

Steilste Stelle ≙ WP Graph g → WP(-28,67 |)

~~(hier kein WP also Rand (bzw f))~~

$g'(-28,67) = 0,62 = 62\%$

$< 65\%$ ✓

b) $g'(-40) = 0$ ✓ waagerechte heisst $g'(x)=0$ hier $g(x)=0$

$\int_{-40}^{} (g(x) - f(x))\,dx \approx 381,74$ FE

$381,74 : 3 \approx 127,55$ FE

$$127,55 = \int_{5}^{40} (g(x) - f(x))\,dx = \left[\frac{1}{1562500}x^5 - \frac{7}{6000}x^3 + 5x\right]_{5}^{40}$$

$$127,55 = 190,87 - \frac{1}{1562500}5^5 + \frac{7}{6000}5^3 - 5b \quad \text{intersect} \quad \boxed{b \approx 13,21\,m}$$

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80 : 3 ≈ 26,67 m

40 − 26,67 m = 13,33 m

13,21 m

1,1 m

c) Ansatz: $p_a''(x) = 0$

$0 = 12ax^2 - 12x \quad | + 12a$

$12a = 12ax^2 \quad | : (12a)$

$1 = x^2 \quad | \pm \sqrt{}$

$\pm 1 = x$

$P_a(\pm 1) = a - 6a + 1$

$= -5a + 1$

$W(\pm 1 \mid -5a + 1)$

$0 = -5a + 1 \quad | -1 \; | \cdot (-\frac{1}{5})$

$a = \frac{1}{5}$

$0 < a < \frac{1}{5}$

$a > 0 \Leftrightarrow 0 < a$

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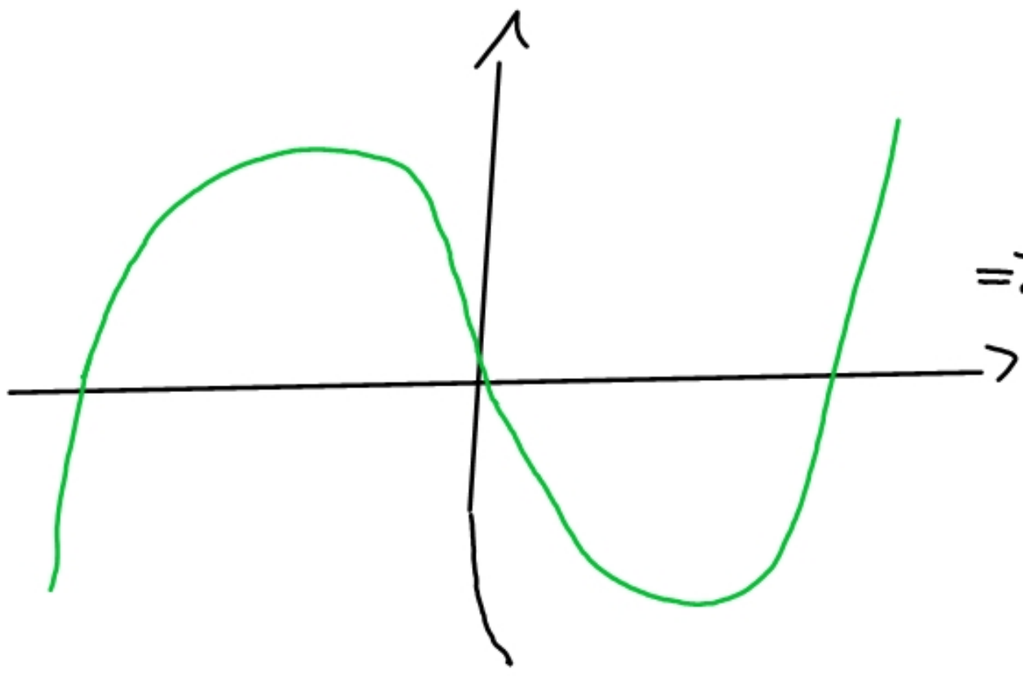
wenn f Achsensymmetrisch ist $\Leftrightarrow$ ist f' punktsymmetrisch $\square$ 0

$\Rightarrow$



$\Rightarrow$ 3 Nullstellen mit VZW von f'

$\hat{=}$ 3 Extremstellen von f. !

Pflichtteil 1. Analysis:

1. Ansatz $f(x) = g(x)$

$4 - x^2 = x^2 - 4 \quad | -4 + x^2$

$0 = \underbrace{2x^2 - 8}_{g(x) - f(x)} \quad | +8 \quad | :2$

$4 = x^2 \quad | \sqrt{}$

$x_{1/2} = \pm 2$

Trick bzw $g(x) - f(x)$
$f(x) - g(x)$

$$\left| \int_{-2}^{2} (2x^2 - 8)\,dx \right| = \left| \left[\tfrac{2}{3}x^3 - 8x \right]_{-2}^{2} \right|$$

$$= \left| \tfrac{2}{3} \cdot 2^3 - 8 \cdot 2 - \left(\tfrac{2}{3}(-2)^3 - 8 \cdot (-2) \right) \right|$$

$$= \left| \tfrac{16}{3} - 16 + \tfrac{16}{3} - 16 \right| = \left| \tfrac{32}{3} - 32 \right|$$

$$= \left| \tfrac{32}{3} - \tfrac{96}{3} \right| = \left| -\tfrac{64}{3} \right| = \tfrac{64}{3}\ \text{FE}$$

(2)

$W(0|0) \longrightarrow f(0) = 0$ I

$\longrightarrow f''(0) = 0$ II

$H(2|2) \longrightarrow f(2) = 2$ III

$f'(2) = 0$ IV

$f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

I $0 = a \cdot 0^3 + b \cdot 0^2 + c \cdot 0 + d$

$0 = d$

II $0 = 6a \cdot 0 + 2b$

$0 = b$

III $2 = 8a + 2c$

IV $0 = 12a +$

HA: 2016 1A + 1B

(3) Ansatz $f(x) = 0$

$0 = e^x (2x + x^2)$

$e^x \neq 0$

$0 = 2x + x^2$

$0 = x(2 + x)$

$x_1 = 0 \qquad x_2 = -2$

b) Ansatz: $F(x) = f(x)$

Produktregel $f = u \cdot v$

$f' = u' \cdot v + u v'$

II $\quad 2 = 8a + 2c$

IV $\quad 0 = 12a + c \quad | \cdot (-2)$

$2 = 8a + 2c$

$0 = -24a - 2c \quad | \oplus$

$2 = -16c \quad | \cdot \left(-\frac{1}{16}\right)$

$-\frac{1}{8} = a$

a in III $\quad 2 = 8 \cdot \left(-\frac{1}{8}\right) + 2c \quad | +1$

$3 = 2c \quad | :2$

$\frac{3}{2} = c$

$\longrightarrow \boxed{f(x) = -\frac{1}{8}x^3 + \frac{3}{2}x}$

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$$F(x) = x^2 \cdot e^x$$

$$F'(x) = 2x e^x + x^2 e^x$$

$$= e^x (2x + x^2) = f(x) \checkmark$$

$$G(x) = F(x) + C$$

$$G(x) = x^2 e^x + C$$

$$G(1) = 2e$$

$$2e = 1^2 \cdot e^1 + C \quad | -e$$

$$e = C$$

$$G(x) = x^2 e^x + e$$

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